Exam 2 Review: Sections 2.1-2.5 and 3.1-3.4

Section 2.1. Consider a rabbit population satisfying the logistic equation

$$\frac{dP}{dt} = 2P - (0.005)P^2.$$

If the initial population is 120 rabbits, how many months does it take for $P(t)$ to reach 95% of its limiting population M?

Section 2.2 Draw the phase diagram for the autonomous differential equation

$$\frac{dx}{dt} = x^2 - 5x + 4$$

and determine which critical points are stable and unstable.

Section 2.3 Consider a body that moves horizontally through a medium whose resistance is proportional to the square of velocity so that

$$\frac{dv}{dt} = -2v^2.$$

Assuming that $v(0) = 1$ and $x(0) = 1$, find the position $x(t)$ as a function of t.

Sections 2.4 Use the Euler method to find an approximation for $y(2)$ using a step size of $h = 0.5$ for the differential equation

$$yy' = 2x^3, \quad y(1) = 3.$$

Section 2.5 Use the Improved Euler method to find an approximation for $y(2)$ using a step size of $h = 0.5$ for the differential equation

$$yy' = 2x^3, \quad y(1) = 3.$$

Sections 3.1-3.3 Find the general form of they solution to the differential equation

$$6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4 = 0$$

which has characteristic function

$$(r^2 + 4)(6r^2 + 5r + 1) = 0.$$

Section 3.4 A 12-lb weight (mass m=0.375 slugs= lbs/g) is attached both to a vertically suspended spring that it stretches 6 in. (thus from lbs-$k \cdot s_0$=0, we get $k = 24$) and to a dashpot that provides 3 lb of resistance for every foot per second of velocity.

(a) The weight is pulled down 1 ft below its static equilibrium position and then released from rest at time $t = 0$, find its position function $x(t)$.

(b) Determine if the motion is over-damped, critically damped or under-damped.